

1) Done in class

2) Let $\dot{x} = y \Rightarrow \dot{y} = 3y - 2x$.

b) $(x_*, y_*) = (0, 0)$. $J(x_*, y_*) = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix}$

$$\begin{vmatrix} -\lambda & 1 \\ -2 & 3-\lambda \end{vmatrix} = \lambda^2 - 3\lambda + 2 = 0 \Rightarrow \lambda = \frac{3 \pm \sqrt{9-8}}{2} = 1, 2$$

c) $\begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix} v_1 = 0 \Rightarrow v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$; $\begin{pmatrix} -2 & 1 \\ -2 & 1 \end{pmatrix} v_2 = 0 \Rightarrow v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

3) $\dot{x} = 2x - x^2 - y$
 $\dot{y} = x - y - z$
 $\dot{z} = y - 2z$

$(x_*, y_*, z_*) = (0, 0, 0), \left(\frac{4}{3}, \frac{8}{9}, \frac{4}{9}\right)$

 \leftarrow
 $y_* = 2z_* \Rightarrow x_* = 3z_* \Rightarrow 6z_* - 9z_*^2 - 2z_* = 0$
 $\Rightarrow z_* (4 - 9z_*) = 0 \Rightarrow z_* = 0, \frac{4}{9}$
 $\Rightarrow y_* = 0, \frac{8}{9}, x_* = 0, \frac{4}{3}$

Can you find the fixed points? yes

What conclusions can you make? Not much w/o more analysis.

4) $\dot{x} = 2x - x^2 - y - 2z$

$$\dot{y} = x - y - 2z$$

$$\dot{z} = y - 2z$$

What can we say about this system? It's certainly more interesting than the previous one.

Can you make a hypothesis before doing any analysis?

I think everything will die off; i.e., $(0, 0, 0)$ is a stable f.p. Did anyone try this out on pplane?