FALL 2015 SOLUTIONS

(1) (a) We solve this by parts, let $u = x \Rightarrow du = dx$, $dv = \sinh 2x \Rightarrow v = \frac{1}{2}\cosh 2x$. The integral becomes, $I = \frac{x}{2}\cosh 2x - \frac{1}{2}\int \cosh 2x dx = \frac{x}{2}\cosh 2x - \frac{1}{4}\sinh 2x + C$.

(b) We solve this by parts, let $u = \sin^{-1} 3x \Rightarrow du = 3dx/\sqrt{1-9x^2}$, $dv = dx \Rightarrow v = x$. The integral becomes, $I = x\sin^{-1} 3x - \int 3x dx/\sqrt{1-9x^2}$. We solve the new integral via u-sub where $u = 1 - 9x^2 \Rightarrow du = -18x dx$,
$I = x\sin^{-1} 3x + \frac{1}{6}\int du/\sqrt{u} = x\sin^{-1} 3x + \sqrt{u}/3 + C = x\sin^{-1} 3x + \sqrt{1-9x^2}/3 + C$.

(2) We get the work for the bucket for free: $W_B = 5 * 50 = 250 ft - lb$. We derive the force function for the water, $F = 40 - x/10$, then we integrate to get the work,
$W_w = \int_0^{50}(40 - x/10)dx = 40x - x^2/20\Big|_0^5 0 = 1875 ft - lb$, so the total work is $W = 2125 ft - lb$.

(3) If we use the coordinate where the bottom of the box is $11ft$ and the top of the box is $1ft$, we have an infinitesimal volume of $V_i = 300\Delta x_i$, then the weight is $F_i = 12000\Delta x_i$, hence the work to move the infinitesimal volume to our height is $W_i = 12000 x\Delta x_i$. Finally, we integrate to get
$W = \int_1^{11} 12000 x dx = 6000x^2\Big|_1^{11} = 720,000 ft - lb$.

(4) The derivative is $dy/dx = -1/2\sqrt{2-x} \Rightarrow (dy/dx)^2 = 1/(8-4x)$, then the surface area is,
$SA = 2\pi\int_0^{5/4}\sqrt{2-x}\sqrt{1 + 1/4(2-x)}dx = \pi\int_0^{5/4}\sqrt{9-4x} = -\frac{\pi}{6}(9-4x)^{3/2}\Big|_0^{5/4} = 19\pi/6$.

(5) The derivative is $dy/dx = \tan x$, then the arc length is,
$L = \int_0^{\pi/4}\sqrt{1+\tan^2 x}dx = \int_0^{\pi/4}\sec x dx = \ln|\tan x + \sec x|\Big|_0^{\pi/4} = \ln(1+\sqrt{2})$.

(6) Here we have a gap, so we need a big radius and a little radius: $R = 2/(x+1)$, $r = x$, then the volume is $V = \pi\int_0^1(4/(x+1)^2 - x^2)dx = \pi\left[-4/(x+1) - x^3/3\right]_0^1 = 5\pi/3$.

(7) Here there is no gap, so $R = 1 - e^{-x}$, then the volume is,
$V = \pi\int_0^1(1-e^{-x})^2 dx = \pi\int_0^1(1-2e^{-x}+e^{-2x})dx = \pi\left[x + 2e^{-x} - e^{-2x}/2\right]_0^1 = \pi\left[-1/2 + 2/e - 1/2e^2\right]$.

(8) Here $r = x$ and $h = \ln x$, then the volume is, $V = 2\pi\int_1^e x\ln x dx$. We solve this by parts, $u = \ln x \Rightarrow du = dx/x$, $dv = x dx \Rightarrow v = x^2/2$. Then the integral becomes,
$V = \pi x^2 \ln x\Big|_1^e - 2\pi\int_1^e (x/2)dx = \pi e^2 - \pi x^2/2\Big|_1^e = \pi e^2/2 + \pi/2$.

(9) Here the area of each rectangle is $A = 4y^2 = 4\sin^2 x$, then the volume is
$V = \int_0^\pi 4\sin^2 x dx = \int_0^\pi 2(1-\cos 2x)dx = 2x - \sin 2x\Big|_0^\pi = 2\pi$.