Exam I Fall 2016:

(1) Disks/Washers: $R = 3$, $r = \ln x$, then

$$V = \pi \int_1^e \left(9 - (\ln x)^2\right) dx$$

Shells: $r = 3 - y$, $h = e - e^y$, then

$$V = 2\pi \int_0^1 (3 - y)(e - e^y) dy$$

(2) $k = F/x = 10/(1/2) = 20lb/ft \Rightarrow F = 20x$, then

$$W = \int_0^1 20x dx = 10x^2\Big|_0^1 = 10ft - lb$$

(3) Shells: $\hat{r} = x$, $\hat{h} = h - hx/r$, then

$$V = 2\pi \int_0^r \left(hx - \frac{h}{r}x^2\right) dx = 2\pi \left[\frac{1}{2}hx^2 - \frac{h}{3r}x^3\right]_0^r = \frac{\pi}{3}hr^2$$

Disks/Washers: $\hat{r} = ry/h$, then

$$V = \pi \int_0^h \frac{r^2}{h^2}y^2 dy = \frac{\pi}{3}\frac{r^2}{h^2}y^3\Big|_0^h = \frac{\pi}{3}hr^2$$

(4) Intersections: $(0, 0)$ and $(2, 4)$

Shells: $r = x$, $h = 2x - x^2$, then

$$V = 2\pi \int_0^2 (2x^2 - x^3) dx = 2\pi \left[\frac{2}{3}x^3 - \frac{1}{4}x^4\right]_0^2 = \frac{8\pi}{3}$$

Disks/Washers: $r = y/2$, $R = \sqrt{y}$, then

$$V = \pi \int_0^4 \left(y - \frac{y^2}{4}\right) dy = \pi \left[\frac{1}{2}y^2 - \frac{1}{12}y^3\right]_0^4 = \frac{8\pi}{3}$$

(5) Here we have to use shells. $r = x$, $h = x^2 - x^3$, then

$$V = 2\pi \int_0^1 (x^3 - x^4) dx = 2\pi \left[\frac{1}{4}x^4 - \frac{1}{5}x^5\right]_0^1 = \frac{\pi}{10}.$$

(6) $u = \sqrt{x} \Rightarrow du = \frac{dx}{2\sqrt{x}}$, then

$$I = 2\int \cos(\sqrt{3}u) du = \frac{2}{\sqrt{3}}\sin(\sqrt{3}u) + C = \frac{2}{\sqrt{3}}\sin(\sqrt{3x}) + C$$

(7)

$$\frac{dx}{dy} = \frac{1}{2}y^2 - \frac{1}{2}y^{-2} \Rightarrow 1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{1}{4}y^4 - \frac{1}{2} + \frac{1}{4}y^{-4} = \frac{1}{4}y^4 + \frac{1}{2} + \frac{1}{4}y^{-4} = (\frac{1}{2}y^2 + \frac{1}{2}y^{-2})^2$$

$$\Rightarrow L = \int_1^2 \left(\frac{1}{2}y^2 + \frac{1}{2}y^{-2}\right) dy = \frac{1}{6}y^3 - \frac{1}{2}y^{-1}\Big|_1^2 = \frac{4}{3} - \frac{1}{4} - \frac{1}{6} + \frac{1}{2} = \frac{17}{2}$$

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(8) $g'(y) = 2y \Rightarrow A = 2\pi \int_0^{\sqrt{2}} y\sqrt{1+4y^2}dy$. We use u-sub, $u = 1+4y^2 \Rightarrow du = 8ydy$, then

$$A = \frac{\pi}{4}\int_1^9 u^{1/2}du = \frac{\pi}{6}u^{3/2}\Big|_1^9 = \frac{9\pi}{2} - \frac{\pi}{6} = \frac{26\pi}{6}$$

(9) Here the area is easy,

$$A_i = \pi r^2 = 4\pi \Rightarrow V_i = 4\pi\delta x_i \Rightarrow F_i = 4\pi 10^4 \delta x_i \Rightarrow W_i = (4\pi 10^4)x_i\delta x_i$$

$$\Rightarrow W = 4\pi 10^4 \int_6^9 xdx = 2\pi 10^4 x^2\Big|_6^9 = 90\pi 10^4$$

Exam II Fall 2016:

1) Here we use by parts with, $u = x \Rightarrow du = dx$ and $dv = e^{-2x}dx \Rightarrow v = -e^{-2x}/2$, then

$$I = -\frac{x}{2}e^{-2x} + \frac{1}{2}\int e^{-2x}dx = -\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x} + C.$$

4) Using $\tan^2 theta = \sec^2\theta - 1$ we get

$$\int \left(\sec^2(x/2) - 1\right)\sec^2(x/2)\left(\tan(x/2)\sec(x/2)\right)dx$$

Then $u = \sec(x/2) \Rightarrow du = (1/2)\sec(x/2)\tan(x/2)dx$, then

$$I = \frac{1}{2}\int (u^4 - u^2)du = \frac{1}{2}\left(\frac{1}{5}u^5 - \frac{1}{3}u^3\right) + C = \frac{1}{2}\left[\frac{1}{5}\sec^5(x/2) - \frac{1}{3}\sec^3(x/2)\right] + C$$

5) This one takes multiple by parts. The first by parts is $u = \cos\pi x \Rightarrow du = -\pi\sin\pi x$ and $dv = e^x dx \Rightarrow v = e^x$, then

$$I = e^x\cos\pi x + \pi\int e^x\sin(\pi x)dx$$

Then we do another by parts, $u = \sin\pi x \Rightarrow du = \pi\cos\pi x$ and $dv = e^x dx \Rightarrow v = e^x$, then

$$I = e^x\cos\pi x + \pi e^x\sin\pi x - \pi^2\int e^x\cos\pi x dx \Rightarrow (1+\pi^2)I = e^x\cos\pi x + \pi e^x\sin\pi x \Rightarrow I = \frac{1}{1+\pi^2}\left(e^x\cos\pi x + \pi e^x\sin\pi\right.$$

8) Here it's a straight u-sub, $u = 4 - x^4 \Rightarrow du = -4x^3dx$, then

$$I = -\frac{1}{4}\int u^{1/3}du = -\frac{3}{16}u^{4/3} + C = -\frac{3}{16}(4-x^4)^{4/3} + C$$

10) Using by parts, $u = \ln x \Rightarrow du = dx/x$ and $dv = dx \Rightarrow v = x$, then

$$I = x\ln x - \int dx = x\ln x - x + C$$