(1) We use two sets of "by parts". First $u = x^2 \Rightarrow du = 2xdx$ and $dv = e^{-x}dx \Rightarrow v = e^{-x}$,

$$I = -x^2e^{-x} + 2\int xe^{-x}dx$$

Next $u = x \Rightarrow du = dx$ and $dv = e^{-x}dx \Rightarrow v = e^{-x}$,

$$I = -x^2e^{-x} - 2xe^{-x} + \int e^{-x}dx = \boxed{-x^2e^{-x} - 2xe^{-x} - e^{-x} + C}$$

(2) (a) We pull out e^3 and do a straight u-sub on $u = e^x + 3 \Rightarrow du = e^x dx$,

$$I = e^3\int \frac{du}{u} = e^3 \ln|u| + C = \boxed{e^3\ln(e^x+3) + C}.$$

(b) We use "by parts", $u = \ln x \Rightarrow du = dx/x$ and $dv = xdx \Rightarrow v = x^2/2$,

$$I = \frac{1}{2}x^2\ln x - \frac{1}{2}\int xdx = \boxed{\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C}.$$

(3) Intersections: $(0,0)$ and $(1,2)$

Washers: $r = x$ and $R = x^{1/4}$,

$$V = \pi\int_0^1 \left(x^{1/2} - x^2\right)dx = \pi\left[\frac{2}{3}x^{3/2} - \frac{1}{3}x^3\right]_0^1 = \boxed{\frac{\pi}{3}}.$$

Shells: $r = y$ and $h = y - y^4$,

$$V = 2\pi\int_0^1 \left(y^2 - y^5\right)dy = 2\pi\left[\frac{1}{3}y^3 - \frac{1}{6}y^6\right]_0^1 = \boxed{\frac{\pi}{3}}.$$

(4) Washers: $R = e + 1$ and $r = 1 + e^y$,

$$\boxed{V = \pi\int_0^1 \left[(e+1)^2 - (1+e^y)^2\right]dy}$$

Shells: $h = \ln x$, $r = x + 1$,

$$\boxed{V = 2\pi\int_1^e (x+1)\ln x dx}.$$

(5) We have to use shells for this one: $r = x$ and $h = \frac{1}{x}\sin^2 x\cos^3 x$, then

$$V = 2\pi\int_0^{\pi/2} \sin^2 x\cos^3 x dx = 2\pi\int_0^{\pi/2} \sin^2 x(1 - \sin^2 x)\cos x dx.$$

Then we use u-sub, $u = \sin x \Rightarrow du = \cos x dx$,

$$V = 2\pi\int_0^1 \left(u^2 - u^4\right)du = 2\pi\left[\frac{1}{3}u^3 - \frac{1}{5}u^5\right]_0^1 = \boxed{\frac{4\pi}{15}}.$$

(6) $y' = 3/x - x/12 \Rightarrow (1 + y'^2) = 9/x^2 - 1/2 + x^2/144 + 1 = 9/x^2 + 1/2 + x^2/144 = (3/x + x/12)^2$,

$$L = \int_1^6 \left(\frac{3}{x} + \frac{x}{12}\right)dx = 3\ln x + \left.\frac{x^2}{24}\right|_1^6 = 3\ln 6 + \frac{36}{24} - \frac{1}{24} = \boxed{3\ln 6 + \frac{35}{24}}.$$

(7) We use $\sinh x = (e^x - e^{-x})/2$,

$$I = \frac{1}{2}\int \left(e^x + e^{-x}\right)\left(e^x - e^{-x}\right)dx = \frac{1}{2}\int \left(e^{2x} - e^{-2x}\right)dx = \boxed{\frac{1}{4}\left[e^{2x} - e^{-2x}\right] + C}.$$

(8) $k = F/x = 10N/(0.1m) = 100N/m \Rightarrow F = 100x$, then

$$W = \int_0^{0.5} 100xdx = 50x^2\Big|_0^{1/2} = \boxed{12.5J}$$

(9) $V_i = \pi r_i^2\Delta x_i = \pi x_i\Delta x_i \Rightarrow F_i = \pi 10^4 x_i\Delta x_i \Rightarrow W_i = \pi 10^4 x_i(5 - x_i)\Delta x_i$,

$$W = \pi 10^4\int_0^2 (5x - x^2)dx = \pi 10^4\left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^2 = \pi 10^4\left[10 - \frac{8}{3}\right] = \boxed{\frac{22}{3}\pi 10^4}.$$