Exam II Fall 2016:

(1) Here we use by parts with, $u = x \Rightarrow du = dx$ and $dv = e^{-2x}dx \Rightarrow v = -e^{-2x}/2$, then

$$I = -\frac{x}{2}e^{-2x} + \frac{1}{2}\int e^{-2x}dx = -\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x} + C.$$

(2) Splitting up the fraction gives us

$$\frac{5x^3 - 3x^2 + 2x - 1}{x^2(x^2+1)} = \frac{Ax+B}{x^2} + \frac{Cx+D}{x^2+1} \Rightarrow (A+C)x^3 + (B+D)x^2 + Ax + B = 5x^3 - 3x^2 + 2x - 1$$

Then we get $A = 2$, $B = -1$, $D = -2$, $C = 3$. So our integral becomes

$$I = 2\int \frac{dx}{x} - \int \frac{dx}{x^2} + \int \frac{3x-2}{x^2+1}dx = 2\ln|x| + \frac{1}{x} + \frac{3}{2}\int \frac{2xdx}{x^2+1} - 2\int \frac{dx}{x^2+1}$$

Now we can do a u-sub on the third integral, $u = x^2 + 1 \Rightarrow du = 2xdx$,

$$I = 2\ln|x| + \frac{1}{x} + \frac{3}{2}\int \frac{du}{u} - 2\tan^{-1}x = 2\ln|x| + \frac{1}{x} + \frac{3}{2}\ln|x^2+1| - 2\tan^{-1}x + C$$

(3) Here partial fractions won't work. Try it, you'll see ;). So we need to use trig-sub, $x = \frac{3}{2}\tan\theta \Rightarrow dx = \frac{3}{2}\sec^2\theta d\theta$. Then

$$I = \int \frac{(3/2)\sec^2\theta d\theta}{9^2\sec^4\theta} = \frac{3}{2\cdot 9^2}\int \cos^2\theta d\theta = \frac{1}{6\cdot 9}\int \frac{1}{2}(1+\cos 2\theta)d\theta = \frac{1}{108}\left[\theta + \frac{1}{2}\sin 2\theta\right]$$

$$= \frac{1}{108}\left[\tan^{-1}\left(\frac{2}{3}x\right) + \sin\theta\cos\theta\right] = \frac{1}{108}\left[\tan^{-1}\left(\frac{2}{3}x\right) + \frac{6x}{4x^2+9}\right]$$

(4) Using $\tan^2 theta = \sec^2\theta - 1$ we get

$$\int \left(\sec^2(x/2) - 1\right)\sec^2(x/2)\left(\tan(x/2)\sec(x/2)\right)dx$$

Then $u = \sec(x/2) \Rightarrow du = (1/2)\sec(x/2)\tan(x/2)dx$, then

$$I = \frac{1}{2}\int (u^4 - u^2)du = \frac{1}{2}\left(\frac{1}{5}u^5 - \frac{1}{3}u^3\right) + C = \frac{1}{2}\left[\frac{1}{5}\sec^5(x/2) - \frac{1}{3}\sec^3(x/2)\right] + C$$

(5) This one takes multiple by parts. The first by parts is $u = \cos\pi x \Rightarrow du = -\pi\sin\pi x$ and $dv = e^x dx \Rightarrow v = e^x$, then

$$I = e^x\cos\pi x + \pi\int e^x\sin(\pi x)dx$$

Then we do another by parts, $u = \sin\pi x \Rightarrow du = \pi\cos\pi x$ and $dv = e^x dx \Rightarrow v = e^x$, then

$$I = e^x\cos\pi x + \pi e^x\sin\pi x - \pi^2\int e^x\cos\pi x dx \Rightarrow (1+\pi^2)I = e^x\cos\pi x + \pi e^x\sin\pi x$$

$$\Rightarrow I = \frac{1}{1+\pi^2}\left(e^x\cos\pi x + \pi e^x\sin\pi x\right)$$

(6)

$$\lim_{n\to\infty}\frac{1-3n}{1+2n} = \lim_{n\to\infty}\frac{1/n - 3}{1/n + 2} = -\frac{3}{2}.$$

(7) Here we can use u-sub if we like, $u = 1 - x^2 \Rightarrow x^2 = 1 - u$ and $du = -2xdx$, then

$$I = -\frac{1}{2}\int \frac{(1-u)du}{\sqrt{u}} = -\frac{1}{2}\int \left(u^{-1/2} - u^{1/2}\right)du = -u^{1/2} + \frac{1}{3}u^{3/2} + C = -\sqrt{1-x^2} + \frac{1}{3}(1-x^2)^{3/2} + C.$$

(8) Here it's a straight u-sub, $u = 4 - x^4 \Rightarrow du = -4x^3dx$, then

$$I = -\frac{1}{4}\int u^{1/3}du = -\frac{3}{16}u^{4/3} + C = -\frac{3}{16}(4-x^4)^{4/3} + C$$

(9) We know what the antiderivative is, so we just have to take the limit

$$\lim_{t\to\infty} \tan^{-1} x\Big|_1^t = \lim_{t\to\infty} \tan^{-1}(t) - \tan^{-1}(1) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$$

(10) Using by parts, $u = \ln x \Rightarrow du = dx/x$ and $dv = dx \Rightarrow v = x$, then

$$I = x \ln x - \int dx = x \ln x - x + C$$

(11) 'tis a silly problem

(a) $x_0 = a$, $x_1 = (a+b)/2$, $x_2 = b$. $\Delta x = (b-a)/2$, $y_0 = ma$, $y_1 = m(a+b)/2$, $y_2 = mb$. Plugging this into the Trapezoid rule formula give us

$$T_2 = \frac{b-a}{4}(ma + m(a+b) + mb) = \frac{m}{2}(b^2 - a^2).$$

(b)

$$\int_a^b mx dx = \frac{m}{2}x^2\Big|_a^b = \frac{m}{2}(b^2 - a^2).$$

(c) $f''(x) = 0$, so $|E_T| = 0$, which is obvious because $y = mx$ is a linear function. You will definitely get a more substantial problem this year!