Math 2450 Rahman Exam I Sample Problems

(1) For a particle moving along the space curve given by $\vec{r} = (2t)\hat{\mathbf{i}} + (1/t)\hat{\mathbf{j}} + (t-1)^3\hat{\mathbf{k}}$
   (a) Evaluate $\vec{r}\,''$ at $t = 1$.

   (b) Determine the curvature at $t = 1$.

(2) If $\vec{a}$ and $\vec{b}$ are constant vectors, calculate

$$\frac{d}{dt}\left[(\vec{a} + t\vec{b}) \times (\vec{b} - t\vec{a})\right].$$

(3) Compute

$$\int_0^{\pi} \left[\left(e^{-t}\right)\hat{\mathbf{i}} - (\sqrt{t})\hat{\mathbf{j}} + (\cos t)\hat{\mathbf{k}}\right] dt.$$

(4) Find a vector and a parametric equation of the line passing through points $P(1, 2, 3)$ and $Q(2, 1, 5)$.

(5) For the curve described by the parametric equations $x = 1 + 2\sin 4t$, $y = 2 + 2\cos 4t$.
   (a) Sketch the curve in the domain $0 \le t \le \pi/8$.
   (b) Determine the equation of the line tangent to the curve at $t = \pi/16$.

(6) What is the area between the curve and the x-axis for the curve described by the parameteric equations $x = t^3 + t^2$, $y = 1/t$, $1 \le t \le 2$?

(7) For the points $P(-1, 0, 2)$, $Q(0, 1, 0)$, and $R(1, 2, 3)$, determine
   (a) The cosine of the angle between the vectors $\vec{RQ}$ and $\vec{PQ}$, and
   (b) The area of the triangle formed by these points.

(8) Consider the plane $x - 2y + z = 2$.
   (a) The parametric equations of the line perpendicular to the given plane through the point $(2, 1, 2)$.
   (b) The equation of the line of intersection of the given plane and the xy-plane.
   (c) The cosine of the angle between the given plane and the xy-plane.

(9) Consider the velocity vector $\vec{v} = (3t^2 + 1)\hat{\mathbf{i}} + (e^t)\hat{\mathbf{j}}$
   (a) The position vector at $t = 1$ if $\vec{r}(0) = \hat{\mathbf{i}} + \hat{\mathbf{j}}$.
   (b) The acceleration vector at $t = 1$.

(10) For the vector $\vec{r} = (t^2)\hat{\mathbf{i}} + (t^3)\hat{\mathbf{j}} + (\cos(t-1))\hat{\mathbf{k}}$
   (a) Find the unit tangent vector at $t = 1$.
   (b) Compute the curvature of the curve at $t = 1$.