Math 2450 Rahman Exam II Sample Problems

(1) Compute the following limits or show it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{xy} \tag{1}$$

$$\lim_{(x,y)\to(1,1)} \frac{(x-1)^2-(y-1)^2}{x-y} \tag{2}$$

$$\lim_{(x,y)\to(0,0)} \cos xy \tag{3}$$

$$\lim_{(x,y)\to(1,1)} \frac{x^2+y^2}{xy} \tag{4}$$

$$\lim_{(x,y)\to(1,0)} \left(\tan^{-1}\frac{x}{y}\right)^2 \tag{5}$$

(2) Find the equation of the tangent plane to the graph of the function $f(x, y) = (x + 2y)\cos(3xy)$ at point $(0, 1, 2)$.

(3) The total resistance R of two resistors with resistance R_1 and R_2 connected in parallel is given by the following well-known formula:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

Suppose that R_1 and R_2 are measured to be 300 and 600 ohms, respectively, with an error of ± 15 ohms in each measurement. Use differentials to estimate the maximum error in ohms in the calculated value of R.

(4) Consider the surface described implicitly by the equation

$$2yx^3 + \frac{z^2}{x} + xy\ln z = 3$$

(a) Find the equation of the tangent plane to the surface at point $(1, 1, 1)$

(b) Using the linear approximation evaluate the approximate value of z on the surface when $x = 1.01$ and $y = 0.98$.

(5) Determine the local extrema locations (critical points) for

$$z = xy^2 + \frac{1}{2}x^2 + y^2 + 10$$

(6) Find and classify (max, min, saddle, inconclusive) the critical points of

$$z = 2(x+1)^2 + 3(y-2)^2 + 6(y-2)$$

(7) Determine the following derivatives, using chain rule, for $w = xe^z + zy$

(a) dw/dt at $t = 1$, where $x = 1/t$, $y = t^3$, and $z = t - 1$

(b) dw/dv at $u = 1$ and $v = 1$, where $x = u^2 + v$, $y = uv^2$, and $z = v^2 - u^2$.

(8) For the surface given by the equation $x^2yz + x + yz^3 = 7$, determine the following at point $(2, 1, 1)$

(a) The equation of the plane tangent to the surface

(b) write $\partial z/\partial x$ only in terms of x, y, z, and constants (just like we did in class).

(9) For the function $f(x, y, z) = x/y + 2xyz$, evaluate the following at point $(1, 1, 0)$

(a) The directional derivative in the direction $\vec{v} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

(b) A unit vector in the direction in which the directional derivative is maximum

(c) The maximum value of the directional derivative

(10) Using Lagrange multipliers, find the point on the plane $x + 2y + 3z = 6$ that is closest to the origin $(0, 0, 0)$.