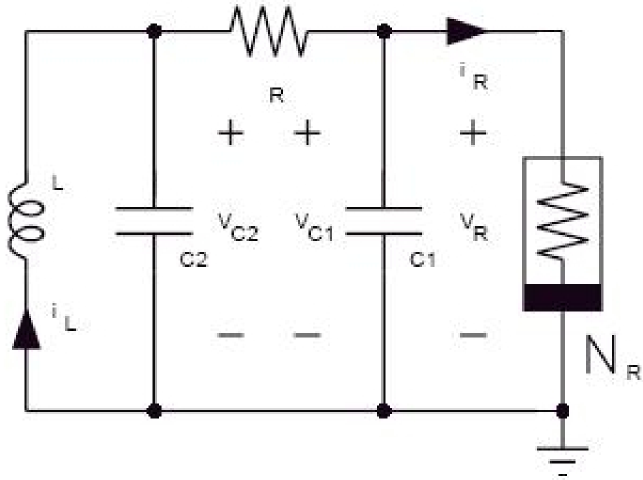
LECTURE ELEVEN: CHUA'S CIRCUIT

Leon Chua designed a circuit in 1983 that exhibits chaos similar to that of the Lorenz Equations. His goal was to design the simplest circuit possible to exhibit chaos.



So we use Kirchoff's voltage and current laws to derive our system of first order ODEs. First lets consider the left most loop, which gives us

$$L\frac{dI_L}{dt} + V_{C_2} = 0. \tag{1}$$

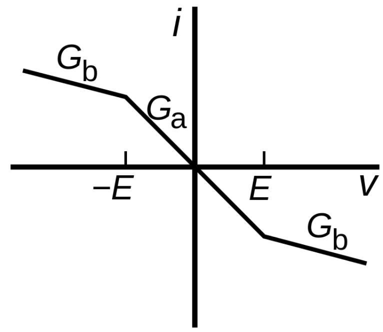
Now if we look at the First nontrivial node on the left we get,

$$C_2\frac{dV_{C_2}}{dt} - \frac{1}{R}\left(V_{C_1} - V_{C_2}\right) - I_L = 0. \tag{2}$$

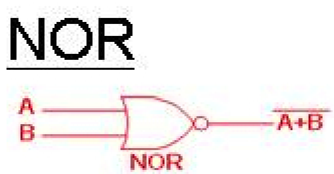
Finally, looking at the right node gives us,

$$C_1\frac{dV_{C_1}}{dt} - \frac{1}{R}\left(V_{C_2} - V_{C_1}\right) + g\left(V_{C_1}\right) = 0. \tag{3}$$

We notice that there is a nonlinear resistor, which behaves significantly differently from a linear resistor. In a linear resistor the current relates to the voltage in a linear manner as $I_R = V/R$, i.e. $g(V) = V/R$. However for the nonlinear resistor g is a nonlinear function. For Chua's original circuit g was a piecewise linear function as shown bellow,

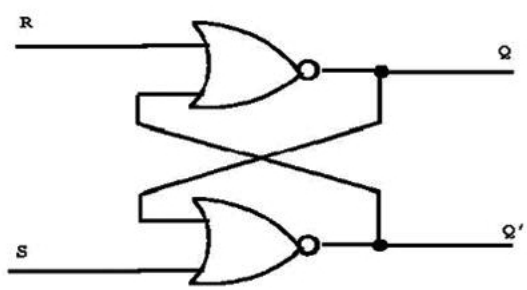


In recent years electrical engineers have been playing around with implementing logical circuits using Chua's circuit. I developed discrete dynamical models for each of the gates and used that to develop a model for the RS flip-flop circuit. The NOR gate behaves as shown in the truth table bellow.



x	y	$F(x,y) := \neg(x \vee y)$
0	0	1
0	1	0
1	0	0
1	1	0

And the RS flip-flop behaves as shown bellow,



R	S	Q	Q'	Operations
0	0	Q	Q'	Keep
0	1	1	0	Set
1	0	0	1	Reset
1	1	Undefined*	Undefined*	Undefined*

We can model the NOR gate by assuming the transition between a "0" state and a "1" state is nonlinear when it is implemented using a Chua circuit. Then we use the model for each NOR gate in the RS flip-flop and get a model for the RS flip-flop.