Math 491 Rahman Midterm Due: Tuesday, March 24, 2015

For these problems, you are encouraged to use pplane to assist in your sketches/intuition, but please sketch by hand and show all work. Also, you are encouraged to use mathematica/wolfram alpha/graphing calculator, but please write down every step that you input into your software and every ouput you get from your software.

(1) Consider the traveling wave ODE for Fischer's equation,

$$u'' + cu' + u(1-u) = 0. \tag{1}$$

(a) (**20 pts.**) For $c = 0$ sketch the phase plane, i.e. do the following steps:
   (i) Convert the second order ODE into a system of two first order ODEs.
   (ii) Find all nullclines.
   (iii) Find all fixed points.
   (iv) Write down the Jacobian for the system.
   (v) Find the stability of the fixed points.
   (vi) Compute the eigenvectors for the fixed points with real eigenvalues only.
   (vii) Sketch the phase plane to the fullest extent, i.e. all nullclines, fixed points, and important trajectories. Make sure these are properly labelled (preferably using different colored pens/pencils).

(b) (**10 pts.**) What happens to the phase plane for $c < 0$ and $c > 0$? (You don't have to be rigorous, but explain your reasoning).

(c) (**20 pts.**) Solve for $c = 0$ using ode45 and plot t vs. u and the phase plane (i.e. u vs. u') for the different types of trajectories that you sketched in part a. (hint: pick a trajectory then pick a point on it with the simplest initial conditions.)

(2) Consider the Henon map,

$$\begin{aligned} x_{n+1} &= 1 - ax_n^2 + y_n, \\ y_{n+1} &= bx_n; \end{aligned} \tag{2}$$

(a) Notice, if $b = 0$ this is a 1-D map: $x_{n+1} = 1 - ax_n^2$.
   (i) (**5 pts.**) Find all fixed points of this case.
   (ii) (**10 pts.**) Find the form of the 2-cycle (do the algebra using mathematica/wolfram alpha).
   (iii) (**10 pts.**) For what values of a does a 2-cycle exist? (do the algebra using mathematica/wolfram alpha)

(b) The Henon map is chaotic for $a = 1.4$ and $b = 0.3$.
   (i) (**7 pts.**) Find the fixed points.
   (ii) (**10 pts.**) Plot the iterates for this (i.e. x_n vs. y_n).
   (iii) (**8 pts.**) Plot the x - time series (i.e. n vs. x_n) and plot the y - time series (i.e. n vs. y_n) on separate plots (preferably subplots).