

Consider the ODE $\ddot{\theta} + \sin \theta = 0$.

- (1) **(1 pt.)** Convert the second order ODE into a system of two first order ODEs.
- (2) **(1 pt.)** Find the nullclines for $-\pi \leq \theta \leq \pi$.
- (3) **(1 pt.)** Find the fixed points for $-\pi \leq \theta \leq \pi$.
- (4) **(2 pt.)** Write down the Jacobian for the system.
- (5) **(6 pt.)** Find the stability of the fixed points.
- (6) **(3 pt.)** Compute the eigenvectors for the fixed points with real eigenvalues only.
- (7) **(6 pt.)** Sketch the phase plane. Don't worry too much about accuracy, just make sure it's correct qualitatively, i.e. just sketch the fixed points and some important trajectories (hint: there will be three different types).