Linear Algebra

- Solving a system of two equations $Ax = b$ (2×2). Feel free to use either Gaussian elimination (linear algebra) or simultaneous equations (regular algebra).

- Know how to take the determinant of a 2×2 matrix.

- Solving 2×2 eigenvalue problems $Ax = \lambda x$: Find the eigenvalues, then find the corresponding eigenvectors.

- Know the three types of eigenvalues for 2×2 matrices:
    - Distinct Real Eigenvalues (i.e. positive discriminant)
    - Repeated Real Eigenvalues (i.e. zero discriminant)
    - Complex Conjugate Eigenvalues (i.e. negative discriminant)

Matrix ODEs

- Convert two coupled ODEs into a single first order 2×2 matrix ODE.

- Solve for the eigenvalues and eigenvectors of a 2×2 matrix ODE.

- Know the general form of solutions for the three cases: Distinct Real, Repeated Real, and Complex Conjugate eigenvalues.

- Solve an IVP.

Dynamical Systems

- Find fixed points.

- Linearize about fixed points (i.e. the first nonconstant term in the Taylor series of the nonlinear function).

- Find the eigenvalues for each linearized matrix. No need to find eigenvectors in this class, although they are important.

- State the stability of each fixed point with proper reasoning.

- Sketch a complete phase portrait: fixed points and important trajectories. No need to find nullclines in this class, although they too are important.

Miscellaneous

- Be fluent with algebraic equations: factor polynomials quickly, root finding, know how to use the discriminant, be able to identify positive and negative regimes of a polynomial (hint: use the number line).

- Recognize the approach to solving a problem before starting any work.

- All the problems, except extra credit, will be similar to homework, notes, and quizzes, but obviously not the same! (i.e. you will have to think a little and make connections).

- It may help to know how to convert a system of ODEs from cartesian to polar, but not necessary.

- There will be three problems. Two of them will be straight computation/sketching. One will be slightly abstract. And the extra credit will be something you have never seen before. The problems will be broken into multiple parts in order to guide you to the solution.