Supplementary problems: 13.3 # 1,3,5,8
Quiz: 13.3

Compulsory problems:
Consider the heat equation

$$\frac{\partial u}{\partial t} = K\frac{\partial^2 u}{\partial x^2} \tag{1}$$

with the following initial and boundary conditions

(1) **[9 pts.]** $u(a,t) = 5$, $u_x(b,t) = 0$. Which of the following initial conditions will yield no solution? Provide reasoning. (**Hint1:** Don't solve the heat equation). (**Hint2:** If the initial temperature profile (initial condition) does not match the boundary conditions or if there is a jump/essential discontinuity at any point in the initial temperature profile, we can't solve the heat equation as is.)

(a) $u(x,0) = 10$

(b) $u(x,0) = 5$

(c) $u(x,0) = \frac{5(x-a)}{a-b} + 5$

(d)
$$u(x,0) = \begin{cases} 6 - \frac{x}{a} & \text{for } a < x < (b-a)/2 \\ 6 - \frac{b-a}{2a} & \text{for } (b-a)/2 < x < b \end{cases}$$

(e)
$$u(x,0) = \begin{cases} 5 & \text{for } a < x < (b-a)/2 \\ 4 & \text{for } (b-a)/2 < x < b \end{cases}$$

Hint3: Here are the three possible reasons that you can give for no solution, otherwise the heat equation will have a solution. Occasionally multiple reasons work for the same problem, but you only need to pick one.

- The initial condition doesn't match the boundary condition at $x = a$.
- The initial condition doesn't match the boundary condition at $x = b$.
- The initial condition has a jump discontinuity in the middle.

(2) **[11 pts.]** $u(1,t) = u(\infty,t) = 0$; $u(x,0) = 1$.
Do a change of variables $x \mapsto \xi$ to put the equation onto a finite domain; i.e. $u(\xi = 0, t) = u(\xi = 1, t)$. And write down the heat equation with this change of variables; i.e. in terms of ξ.
Hint: Recall if $\xi = f(x)$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \xi}\frac{\partial \xi}{\partial x} = f'(x)\frac{\partial u}{\partial \xi} \tag{2}$$

(3) **[40 pts.]**

$$u(0,t) = u_x(2,t) = 0; \qquad u(x,0) = \begin{cases} x & \text{for } 0 < x < 1 \\ 1 & \text{for } 1 < x < 2 \end{cases} \tag{3}$$

Solve the heat equation and write down the complete solution. You can skip the nonessential steps, but please show the integration.

Your homework raw score is: $\frac{n}{2m} \cdot M + \left(1 - \frac{n}{2m}\right) \cdot N = N + \frac{n}{2m}(M - N)$. For this homework, $M = 60$, $m = 4$, N is the number of compulsory problems you get correct, and n is the number of supplementary problems you complete. It should be noted that for the supplementary problems I will be looking for **full completion**, but I won't take off points for mistakes.