

Supplementary problems: 13.3 # 1,3,5,8

Quiz: 13.3

Compulsory problems:

Consider the heat equation

$$\frac{\partial u}{\partial t} = K \frac{\partial^2 u}{\partial x^2} \quad (1)$$

with the following initial and boundary conditions

- (1) [9 pts.]  $u(a, t) = 5$ ,  $u_x(b, t) = 0$ . Which of the following initial conditions will yield no solution? Provide reasoning.

(**Hint1:** Don't solve the heat equation). (**Hint2:** If the initial temperature profile (initial condition) does not match the boundary conditions or if there is a jump/essential discontinuity at any point in the initial temperature profile, we can't solve the heat equation as is.)

(a)  $u(x, 0) = 10$

(b)  $u(x, 0) = 5$

(c)  $u(x, 0) = \frac{5(x-a)}{a-b} + 5$

(d)

$$u(x, 0) = \begin{cases} 6 - \frac{x}{a} & \text{for } a < x < (b-a)/2 \\ 6 - \frac{b-a}{2a} & \text{for } (b-a)/2 < x < b \end{cases}$$

(e)

$$u(x, 0) = \begin{cases} 5 & \text{for } a < x < (b-a)/2 \\ 4 & \text{for } (b-a)/2 < x < b \end{cases}$$

**Hint3:** Here are the three possible reasons that you can give for no solution, otherwise the heat equation will have a solution. Occasionally multiple reasons work for the same problem, but you only need to pick one.

- The initial condition doesn't match the boundary condition at  $x = a$ .
- The initial condition doesn't match the boundary condition at  $x = b$ .
- The initial condition has a jump discontinuity in the middle.

- (2) [11 pts.]  $u(1, t) = u(\infty, t) = 0$ ;  $u(x, 0) = 1$ .

Do a change of variables  $x \mapsto \xi$  to put the equation onto a finite domain; i.e.  $u(\xi = 0, t) = u(\xi = 1, t)$ . And write down the heat equation with this change of variables; i.e. in terms of  $\xi$ .

**Hint:** Recall if  $\xi = f(x)$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \xi} \frac{\partial \xi}{\partial x} = f'(x) \frac{\partial u}{\partial \xi} \quad (2)$$

- (3) [40 pts.]

$$u(0, t) = u_x(2, t) = 0; \quad u(x, 0) = \begin{cases} x & \text{for } 0 < x < 1 \\ 1 & \text{for } 1 < x < 2 \end{cases} \quad (3)$$

Solve the heat equation and write down the complete solution. You can skip the nonessential steps, but please show the integration.

Your homework raw score is:  $\frac{n}{2m} \cdot M + \left(1 - \frac{n}{2m}\right) \cdot N = N + \frac{n}{2m}(M - N)$ . For this homework,  $M = 60$ ,  $m = 4$ ,  $N$  is the number of compulsory problems you get correct, and  $n$  is the number of supplementary problems you complete. It should be noted that for the supplementary problems I will be looking for **full completion**, but I won't take off points for mistakes.