(1) Sketch the direction field for $y' = y(y^2 - 4)$ and state what happens as $t \to \infty$.

(2) Sketch the direction field for $y' = y(y-2)^2$.

(3) Consider the ODE $2t^2 y'' + 3ty' - y = 0$.
  (a) What is the order of the equation? Is it linear or nonlinear?
  (b) Is $y_1 = t^{1/2}$ a solution?
  (c) Is $y_2 = t^{-1}$ a solution?

(4) What is the order of the ODE $\frac{d}{dx}\left(x\frac{dy}{dx}\right) = \frac{\ln x}{xy}$. Is it linear or nonlinear?

(5) Solve the IVP $\frac{dy}{dx} = \frac{x}{y(1+x^2)}$, $y(0) = -2$.

(6) Solve the IVP $y' = y^2 - 1$, $y(0) = -2$.

(7) Solve the IVP $y' + y = e^{-t}$, $y(0) = y_0$. Find the value of y_0 such that the solution $y(t)$ reaches its maximum at $t = 4$.

(8) Solve the IVP $ty' + 2y = 4t^2$, $y(1) = 4$.

(9) Use Euler's method to approximate the solution of $y' = -y + 1 - t$, $y(0) = 1$ at $t = 0.2$, for $h = 0.1$.

(10) Find a linear homogeneous constant coefficient ODE that has the roots of its characteristic equation as $r = -2, 3$.

(11) Solve the IVP $y'' - y' - 2y = 0$; $y(0) = \alpha$, $y'(0) = \beta$. What relation between α and β will give us a bounded (for all time) solution?

(12) A tank with a capacity of 6 L initially contains 10 g of salt and 1 L of water. A mixture containing 1 g/L of salt enters the tank at a rate of 3 L/hr. The mixture leaves the tank at a rate of $\frac{1}{2}V(t)$ L/hr, where $V(t)$ is the volume of fluid in the tank (which may be less than the volume of the tank itself).
  (a) Formulate the IVP for the volume of fluid in the tank then solve that IVP.
  (b) Formulate an ODE for the amount of salt in the tank. Show that this ODE is the same as the ODE for the volume above.

(13) A tank initially contains 120 L of fresh water. A mixture containing a concentration of γ g/L of salt enters the tank at a rate of 2 L/min, and the well-stirred mixture leaves the tank at a rate of 3 L/min.
  (a) When will the tank be empty (i.e. come up with an IVP for the volume).
  (b) Formulate an IVP for the amount of salt in the tank.