(1) Consider the IVP: $t(t-4)y'' - 3ty' + 4y = 2;\ y(3) = 0,\ y'(3) = -1$.
  (a) Please determine the longest interval for which the IVP is guaranteed to have a unique solution.
  (b) Please compute the Wronskian using Abel's theorem.

(2) Consider the IVP: $4y'' + 12y' + 9y = 0;\ y(0) = -1,\ y'(0) = \alpha$.
  (a) For what $\alpha$ does the solution change signs at $t = 1/2$?
  (b) How many times does this solution (for the $\alpha$ above) change signs for $t > 0$?

(3) One solution to $t^2y'' - 3ty' + 4y = 0$ is $y_1 = t^2$. Please find the other solution.

(4) One solution to $2t^2y'' + 3ty' - y = 0$ is $y_1 = 1/t$. Please find the other solution.

(5) Please use the method of undetermined coefficients to find the form of the particular solution (**WITHOUT SOLVING FOR CONSTANTS**) of the following ODEs.
  (a)
$$y'' + 5y' + 6y = -t + e^{-3t} + te^{-2t} + e^{-3t}\cos t$$
  (b)
$$y'' + 3y' + 2y = e^t(t^2+1)\sin(2t) + 3e^{-t}\cos t + 4e^t.$$

(6) Please find the general solution of the ODE: $y'' + 4y' + 4y = t^{-2}e^{-2t};\ t > 0$

(7) Consider the ODE $y'' + 2y' + 2y = \cos t$.
  (a) Please find the general solution.
  (b) What happens to the solution as $t \to \infty$?

(8) Consider the ODE $2t^2y'' - ty' + y = t\sqrt{t}$.
  (a) One solution to the homogeneous ODE is $y_1 = t$. Use reduction of order to find the other solution $y_2$.
  (b) Use the characteristic solution $y_c = c_1y_2 + c_2y_2$ to find the general solution to the full ODE.

(9) Please solve the IVP: $y'' + 4y = 6\sin(4t);\ y(0) = y'(0) = 0$.

(10) Consider the IVP $y'' - 3y' - 4y = t + 2;\ y(0) = 3,\ y'(0) = 0$.
  (a) Please find the solution to the IVP.
  (b) What happens to the solution as $t \to \infty$?

(11) A mass weighing 1/2 lb (i.e. mass $= 1/64 lb \cdot s^2/ft$) stretches a spring 1/2 ft.
  (a) Suppose the system has no damping. The mass is initially pulled down 1/2 ft and released.
    (i) Write down the IVP for this system.
    (ii) Solve the IVP.
    (iii) When does the mass return to the equilibrium position (i.e. $x = 0$).
  (b) Now suppose the system has a damping constant of $2 lb \cdot s/ft$. The mass is initially pushed up 1/2 ft and released with a downward velocity of 1/2 ft/s.
    (i) Write down the IVP for this system.
    (ii) Solve the IVP.

(12) Given that $y_1 = 1/t$ is a solution, please find another solution to the ODE
$$t^2y'' + 3ty' + y = 0; t > 0$$

(13) Please solve the following IVP
$$y'' + 4y = 3\sin 2t;\ y(0) = 2,\ y'(0) = -1.$$

# Brief Summary of Chapter 3

**Obviously incomplete so make sure you read the notes as well!**

- Section 3.2: Existence and Uniqueness and Wronskian ($y'' + p(x)y' + q(x)y = 0;\ y(0) = y_0,\ y'(0) = Y_0$)
  - Existence and Uniqueness: Put ODE in standard form and list intervals for which the coefficients and forcing function are continuous, then pick out the interval that contains the initial conditions.
  - Wronskian: $W(y_1, y_2) = y_1 y_2' - y_2 y_1'$.
  - Abel's Theorem: $W(y_1, y_2) = C \exp\left(-\int^x p(\xi) d\xi\right)$

- Section 3.3: Complex Roots: $r = \xi \pm i\theta \Rightarrow y = e^{\xi x}(A\cos\theta x + B\sin\theta x)$.

- Section 3.4: Repeated Roots and Reduction of Order
  - Repeated Roots: $y = (c_1 + c_2 t)e^{rt}$.
  - Reduction of Order: If $y_1$ is a solution, let $y = v(x)y_1$, plug it into the ODE. The $v(x)$ term disappears and you're left with $v'$ and $v''$ terms. Let $u = v'$, solve that ODE, then integrate $u$ to get $v$. If you kept the constants of integration multiply $v$ by $y_1$ to get your general solution.

- Section 3.5: Undetermined coefficients: Find $y_c$. Use $f(x)$ to guess at $y_p$ and group like terms. If there are no repeats, that's your $y_p$. If there are repeats, get rid of the repeats by multiplying through by $x$ as many times as needed.

- Section 3.6: Variation of Parameters: If we know $y_1$ and $y_2$ (sometimes given, sometimes found by solving the homogeneous equation), then $y = -y_1 \int \frac{y_2 f(x)}{W(y_1,y_2)} dt + y_2 \int \frac{y_1 f(x)}{W(y_1,y_2)} dt$.

- Section 3.7: Applications without forcing: Know how to set up and solve the IVPs. The amplitude in undamped oscillatory motion is the constant: $R = \sqrt{A^2 + B^2}$. For damped oscillatory motion $R(t) = e^{\xi t}\sqrt{A^2 + B^2}$. For undamped oscillatory motion the frequency and period are $\omega = \sqrt{k/m}$ and $T = 2\pi/\omega$. For damped oscillatory motion the quasi-frequency and quasi-period are $\theta$ (the imaginary part of the root) and $T_d = 2\pi/\theta$.

- Section 3.8: Applications with forcing: Know your trig identities. The transient solution is the one that goes to zero. The steady state solution is the one that persists. You can get resonance in an undamped system if the frequency of your forcing function is the same as the frequency of your characteristic solution because this causes a repeat and the particular solution has to be multiplied by $t$. You can get resonance in an undamped system if damping is low enough and the frequency of your forcing function is close enough to the "natural frequency", $\omega_0$ (the frequency of the system in the absence of forcing and damping).

  Important identities: $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$ and $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$. Typically, $a = \frac{1}{2}(\omega_0 + \omega)t$ and $b = \frac{1}{2}(\omega_0 - \omega)t$ for this section.

- Trig/Hyp identities that may be useful overall:

$$\sin^2\theta + \cos^2\theta = 1;\ \cosh^2 x - \sinh^2 x = 1$$
$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b;\ \sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$
$$\sin^2\frac{\theta}{2} = \frac{1}{2}(1 - \cos 2\theta);\ \cos^2\frac{\theta}{2} = \frac{1}{2}(1 + \cos 2\theta)$$

You can basically derive any other (seldom used) trig identity that you may need from these by either dividing through by a sin or a cos.