(1) (a) $y = x^r \Rightarrow y' = rx^{r-1} \Rightarrow y'' = r(r-1)x^{r-2}$. Plugging this into the ODE gives us $r(r-1)x^r - 2x^r = (r^2 - r - 2)x^r = 0$, so $r^2 - r - 2 = 0 \Rightarrow \boxed{r = 2, -1}$, then our characteristic solution is $\boxed{y_c = c_1x^2 + c_2x^{-1}}$.

(b) From the characteristic solution we compute our Wronskian,

$$W(y_1, y_2) = \begin{vmatrix} x^2 & x^{-1} \\ 2x & -x^{-2} \end{vmatrix} = -1 - 2 = \boxed{-3}$$

Don't forget to put the equation into standard form, which gives us $\boxed{f(x) = 3/x^3 + 1}$. Then we plug into our variation of parameters formula to get

$$y = -x^2 \int \frac{x^{-1}\left(\frac{3}{x^3} + 1\right)}{-3} dx + x^{-1} \int \frac{x^2\left(\frac{3}{x^3} + 1\right)}{-3} dx = -x^2 \int \left(-x^{-4} - \frac{1}{3}x^{-1}\right) dx + x^{-1} \int \left(-x^{-1} - \frac{1}{3}x^2\right) dx$$

$$= -x^2\left(\frac{1}{3}x^{-3} - \frac{1}{3}\ln x + c_3\right) + x^{-1}\left(-\ln x - \frac{1}{9}x^3 + c_4\right) = \boxed{-\frac{1}{3x} + \frac{1}{3}x^2 \ln x - c_3x^2 - \frac{1}{x}\ln x - \frac{1}{9}x^2 + c_4\frac{1}{x}}$$

(2) (a) $y = vy_1 \Rightarrow y' = v'y_1 + vy_1' \Rightarrow y'' = v''y_1 + 2v'y_1' + vy_1''$.

$$(t-1)y'' - ty' + y = (t-1)\left[v''y_1 + 2v'y_1' + vy_1''\right] - t\left[v'y_1 + vy_1'\right] + vy_1 = (t-1)\left[e^tv'' + 2e^tv'\right] - te^tv' = 0$$

$$= (t-1)y_1v'' + [2(t-1)y_1' - ty_1]v' + [\cancel{(t-1)y_1'' - ty_1' + y_1}]v = 0 \quad (\to 0)$$

$$\Rightarrow (t-1)e^tv'' + \left[2e^t(t-1) - te^t\right]v' = (t-1)e^tv'' + (t-2)e^tv' = 0 \Rightarrow v'' = -\frac{t-2}{t-1}v'.$$

Let $u = v'$,

$$u' = \left[\frac{1}{t-1} - 1\right]u \Rightarrow \int \frac{du}{u} = \int \left[\frac{1}{t-1} - 1\right] dt \Rightarrow \ln|u| = \ln|t-1| - t + C_0 \Rightarrow \boxed{u = k(t-1)e^{-t}} \Rightarrow \boxed{v = k\int(1-t)e^{-t}}.$$

By parts: $u = t - 1 \Rightarrow du = dt$ and $dv = e^{-t} \Rightarrow v = -e^{-t}$,

$$v = k\left[(1-t)e^{-t} + \int e^{-t}dt\right] = k\left[(1-t)e^{-t} - e^{-t} + C_1\right] \Rightarrow \boxed{y = k_1t + C_2e^t} \Rightarrow \boxed{y_2 = t}.$$

(b) $W(y_1, y_2) = C\exp\left(-\int^t q_1(s)ds\right)$.

$$-\int^t q_1(s)ds = \int^t \frac{s}{s-1}ds = \int^t \frac{(s-1)+1}{s-1} = \int^t 1 + \frac{1}{s-1} = \ln|t-1| + t \Rightarrow W(y_1, y_2) = \boxed{C(t-1)e^t}.$$

(c) Put it into standard form:

$$(t-1)y'' - ty' + y = \ln(t)\tan(t) \Rightarrow y'' - \frac{t}{t-1}y' + \frac{1}{t-1}y = \frac{\ln(t)\tan(t)}{t-1}.$$

The discontinuities are at $t = 0, 1, \pm n\pi/2$, however the initial condition is at $t = 1/2$, so the only possible interval is $t \in \boxed{(0,1)}$.

(3) $r^2 - 10r + 25 = (r-5)^2 = 0 \Rightarrow \boxed{r = 5} \Rightarrow \boxed{y = (c_1 + c_2x)e^{5x}}$. Our guess for the particular solution is $y_p \stackrel{?}{=} Ae^{5x}$, however we have a repeat with repeated roots, so our particular solution actually is

$$\boxed{y_p = Ax^2e^{5x}} \Rightarrow y_p' = 5Ax^2e^{5x} + 2Axe^{5x} \Rightarrow y_p'' = 25Ax^2e^{5x} + 20Axe^{5x} + 2Ae^{5x}$$

$$\Rightarrow 25Ax^2e^{5x} + 20Axe^{5x} + 2Ae^{5x} - 50Ax^2e^{5x} - 20Axe^{5x} + 25Ax^2e^{5x} = 2Ae^{5x} = 10e^{5x}.$$

Then, $\boxed{A = 5}$, so the particular solution is $\boxed{y_p = 5x^2e^{5x}}$, and the general solution is $\boxed{y = (c_1 + c_2x + 5x^2)e^{5x}}$. Plugging in the initial conditions gives us $y(0) = \boxed{c_1 = 0} \Rightarrow y = (c_2x + 5x^2)e^{5x}$. For the second initial condition we take the derivative: $y' = (c_2 + 10x)e^{5x} + 5(c_2x + 5x^2)e^{5x}$, then $y'(0) = \boxed{c_2 = 1}$, so the solution is $\boxed{y = (x + 5x^2)e^{5x}}$.

(4) (a) $y_c = A_1\cos 2t + A_2\sin 2t$. Our guess at the particular solution is

$$y_p \overset{?}{=} (a_2t^2 + a_1t + a_0)(B_1\cos 2t + B_2\sin 2t) + (b_1t + b_0)(D_1\cos 2t + D_2\sin 2t)$$

We have repeats with the sines and cosines, and since it didn't tell us to simplify or solve for anything I will just multiply both blocks of terms by t and leave it as is,

$$y_p = t(a_2t^2 + a_1t + a_0)(B_1\cos 2t + B_2\sin 2t) + t(b_1t + b_0)(D_1\cos 2t + D_2\sin 2t)$$

(b) $\boxed{y_c = c_1 + c_2e^{-2t}}$. Our guess for the particular solution is $y_p \overset{?}{=} a_0 + a_1e^{-2t}$, so we have a repeat on both terms, then it becomes

$$y_p = a_0t + a_1te^{-2t} \Rightarrow y_p' = a_0 + a_1e^{-2t} - 2a_1te^{-2t} \Rightarrow y_p'' = -4a_1e^{-2t} - 4a_1te^{-2t}$$
$$\Rightarrow -4a_1e^{-2t} + 4a_1te^{-2t} + 2a_0 + 2a_1e^{-2t} - 4a_1te^{-2t} = 2a_0 - 2a_1e^{-2t} = 3te^{-2t}$$

Then $\boxed{a_0 = 3/2}$ and $\boxed{a_1 = -1/2}$. So the general solution is $\boxed{y = c_1 + c_2e^{-2t} + \frac{3}{2}t - \frac{1}{2}te^{-2t}}$. The first initial condition gives us $y(0) = c_1 + c_2 = 0 \Rightarrow \boxed{c_1 = -c_2}$. Plugging it back in gives $y = c_1 - c_1e^{-2t} + \frac{3}{2}t - \frac{1}{2}te^{-2t} \Rightarrow y' = 2c_1e^{-2t} + \frac{3}{2} - \frac{1}{2}e^{-2t} + te^{-2t}$. Plugging in the second initial condition gives $y'(0) = 2c_1 + 1 = 0 \Rightarrow \boxed{c_1 = -1/2}$ and we get $\boxed{y = -\frac{1}{2} + \frac{1}{2}e^{-2t} + \frac{3}{2}t - \frac{1}{2}te^{-2t}}$

(5) $m = 1/2$, $k = 16/(8/17) = 34$, $\gamma = 2$, then our IVP is

$$\boxed{\frac{1}{2}u'' + 2u' + 34u = 0;\ u(0) = 0,\ u'(0) = -\frac{1}{2}}.$$

We have $r = -2 \pm \sqrt{4 - 68} = \boxed{-2 \pm 8i} \Rightarrow \boxed{u = e^{-2t}[A\cos 8t + B\sin 8t]}$. Plugging in the first initial condition gives $u(0) = \boxed{A = 0}$. The derivative is $u' = -2e^{-2t}B\sin 8t + 8Be^{-2t}\cos 8t$. The second initial condition gives us $u'(0) = 8B = -1/2 \Rightarrow B = -1/16$. Our solution is $\boxed{u = -\frac{1}{16}e^{-2t}\sin 8t}$.

(6) $m = 1/8$, $k = 4/((3/2)/12) = 32$, then our IVP is

$$\boxed{\frac{1}{8}u'' + 32u = 3\cos 15t;\ u(0) = u'(0) = 0}.$$

Then we get $(1/8)r^2 + 32 = 0 \Rightarrow r^2 = -2^2 \cdot 2 \cdot 2 \cdot 4^2 \Rightarrow \boxed{r = \pm 16i}$. So, the characteristic solution is $\boxed{u_c = A_1\cos 16t + A_2\sin 16t}$. We will see that there won't be any repeats, so our particular solution is

$$u_p = B_1\cos 15t + B_2\sin 15t \Rightarrow u_p'' = -15^2B_1\cos 15t - 15^2B_2\sin 15t$$
$$\Rightarrow -\frac{15^2}{8}B_1\cos 15t - \frac{15^2}{8}B_2\sin 15t + 32B_1\cos 15t + 32B_2\sin 15t$$
$$= \left(32 - \frac{15^2}{8}\right)B_1\cos 15t + \left(32 - \frac{15^2}{8}\right)B_2\sin 15t = 3\cos 15t.$$

Then $\boxed{B_2 = 0}$ and $B_1 = 3/(16^2 - 15^2) = \boxed{3/31} \Rightarrow \boxed{u_p = (3/31)\cos 15t}$. So the general solution is $\boxed{u = A_1\cos 16t + A_2\sin 16t + (3/31)\cos 15t}$. The initial conditions give us $u(0) = A_1 + 3/31 = 0 \Rightarrow \boxed{A_1 = -3/31}$ and $u'(0) = 16A_2 = 0$, then our solution is $\boxed{u = \frac{3}{31}[\cos 15t - \cos 16t]}$. However in order to plot it we need to use our trig identities with the definitions: $\boxed{a = 31t/2}$ and $\boxed{b = t/2}$, then

$$\cos 15t = \cos(a - b) = \cos a\cos b + \sin a\sin b;\ \cos 16t = \cos(a + b) = \cos a\cos b - \sin a\sin b$$
$$\Rightarrow \cos 15t - \cos 16t = 2\sin a\sin b = 2\sin\frac{31}{2}t\sin\frac{1}{2}t$$

Now our solution is in a form that can be plotted: $\boxed{u = \frac{6}{31}\sin\frac{32}{2}t\sin\frac{1}{2}t}$