(1) Find α such that $y_1 = x^{1/2}$ is a solution to the ODE

$$x^2y'' + \alpha xy' + y = 0$$

and find the other linearly independent solution y_2 (hint: it's easier than it looks).

(2) Find all singular points and determine their regularity for the following ODE

$$(1 - x^2)y'' - 2xy' + \beta(\beta + 1)y = 0.$$

(3) Consider the power series solution to the ODE $y'' + y = 0$.
   (a) Find the recurrence relation.
   (b) Write out the first two nonzero terms for the two linearly independent solutions.
   (c) Determine the radius of convergence for each series by using the ratio test.

(4) Consider the function

$$g(t) = \begin{cases} e^{-t} & 0 \le t < 1 \\ e^{-3t} + 1 & 1 \le t < 2 \\ 1 & t \ge 2 \end{cases}$$

   (a) Graph the function for $0 \le t \le 3$.
   (b) Write $g(t)$ as unit step functions; i.e. the "u" notation.
   (c) Find the Laplace transform of the function.

(5) Solve the following IVP

$$y'' + 4y' + 8y = 2u_\pi(t) - 2\delta(t - 2\pi);\ y(0) = 2,\ y'(0) = 0$$

(6) Use a convolution to find the Laplace Transform of (Don't integrate)

$$F(s) = \frac{1}{s^3(s^2 + 1)}.$$

(7) Find the Laplace Transform of

$$f(t) = \int_0^t (t - \tau)^2 \cos(2\tau) d\tau$$

(8) Find the inverse Laplace Transform (in closed form) of

$$F(s) = \frac{s^2 - 9}{s^3 + 6s^2 + 9s}.$$

(9) Find the inverse Laplace Transform (in closed form) of

$$G(s) = e^{-s}\frac{s - 2}{s^2 + 2s + 2}$$

(10) Find the inverse Laplace Transform of

$$F(s) = \frac{3}{s^2 + 4}$$

(11) Find the inverse Laplace Transform of

$$F(s) = \frac{2s - 3}{s^2 - 4}$$

(12) Find the inverse Laplace Transform of

$$F(s) = \frac{1 - 2s}{s^2 + 2s + 10}$$

(13) Use Laplace Transforms to solve the IVP

$$y^{(4)} - y = 0;\ y(0) = 1,\ y'(0) = 0,\ y''(0) = -2,\ y'''(0) = 0.$$

(14) Use Laplace Transforms to solve the IVP

$$y'' + 4y' = \begin{cases} t & 0 \leq t < 1, \\ 0 & t \geq 1 \end{cases};\ y(0) = y'(0) = 0$$

(15) Use Laplace Transforms to solve the IVP

$$y' + ay = e^{\lambda t};\ y(0) = c,$$

with $a \neq 0$. What happens to the solution when $\lambda + a \neq 0$? What about for $\lambda + a = 0$?