Suggested problems: Sec. 2.1 # 6c, 9c, 17, 19, 22(b,c), 27, 28, 31, 33, 34, 35; Sec. 2.3 # 2, 4, 7, 9, 16, 18a; Sec. 2.7 # 2, 18, 19; Sec. 3.1 # 3, 6, 8, 10, 13, 17, 20, 22.

Mandatory problems:

(1) Consider the IVP, where b is a constant,

$$y' = -y + be^{-t};\ y(0) = 0.$$

(a) **[5 pts.]** Solve the IVP.
(b) **[2 pts.]** Show that the solution attains its maximum value at $t = 1$.
(c) **[2 pts.]** For what value of b is this maximum $y = 2$?

(2) Consider the IVP, where a is a constant,

$$ty' + (t+1)y = 2te^{-t},\ t > 0;\ y(1) = a.$$

(a) **[6 pts.]** Solve the IVP.
(b) **[1 pts.]** Show that the solution $y \to 0$ as $t \to \infty$
(c) **[3 pts.]** If $y = 0$ at $t = 2$, what is a?
(d) **[3 pts.]** If the solution y has a critical point at $t = 1/2$, what is a?

(3) Consider two connected tanks: Tank 1 and Tank 2. Initially Tank 1 contains 100 gal of fresh water and Tank 2 100 gal of brine containing 10 lb of salt. Brine containing 0.5 lb/gal of salt is pumped into Tank 1 at 1 gal/min, and the mixture leaves Tank 1 and into Tank 2 and finally out of Tank 2 at the same rate.
(a) **[5 pts.]** Derive the IVP (i.e. ODE + IC) for the salt content in Tank 1.
(b) **[5 pts.]** Derive the IVP for the salt content in Tank 2.
(c) **[4 pts.]** Find the amount of salt in Tank 1 for any time (i.e. solve the IVP).
(d) **[6 pts.]** Find the amount of salt in Tank 2 for any time.

(4) Consider the IVP $y' = 2y - 1;\ y(0) = 1$.
(a) **[5 pts.]** Find the exact solution (i.e. solve the IVP) for $t = 0.2$.
(b) **[10 pts.]** Use Euler's method to approximate the solution for $h = 0.1, 0.05, 0.01$.
(c) **[3 pts.]** Find the error (i.e. difference between approximate solution and exact solution) for each h.
(d) **[1 pts.]** Is this error decreasing as h decreases?

(5) Consider the IVP

$$2y'' + 3y' - 2y = 0;\ y(0) = 1,\ y'(0) = -\beta \text{ (with } \beta > 0).$$

(a) **[7 pts.]** Solve the IVP.
(b) **[7 pts.]** Plot the solution for $\beta = 1$.
(c) **[4 pts.]** Find the minimum of the solution.
(d) **[2 pts.]** Find the smallest (in magnitude) value for β for which the solution has no minimum.

Your homework raw score is: $\dfrac{n}{2m} \cdot M + \left(1 - \dfrac{n}{2m}\right) \cdot N = N + \dfrac{n}{2m}(M - N)$.