

- (1) Please use the method of undetermined coefficients to find the form of the particular solution (**WITHOUT SOLVING FOR CONSTANTS**) of the following ODEs.

(a)

$$y'' + 5y' + 6y = -t + e^{-3t} + te^{-2t} + e^{-3t} \cos t$$

(b)

$$y'' + 3y' + 2y = e^t(t^2 + 1) \sin(2t) + 3e^{-t} \cos t + 4e^t.$$

- (2) Please find the general solution of the ODE: $y'' + 4y' + 4y = t^{-2}e^{-2t}$; $t > 0$

- (3) Consider the ODE $y'' + 2y' + 2y = \cos t$.

(a) Please find the general solution.

(b) What happens to the solution as $t \rightarrow \infty$?

- (4) Please solve the IVP: $y'' + 4y = 6 \sin(4t)$; $y(0) = y'(0) = 0$.

- (5) Consider the IVP $y'' - 3y' - 4y = t + 2$; $y(0) = 3$, $y'(0) = 0$.

(a) Please find the solution to the IVP.

(b) What happens to the solution as $t \rightarrow \infty$?

- (6) Consider the ODE $2t^2y'' - ty' + y = t\sqrt{t}$.

(a) Verify the solutions to the homogeneous ODE are $y_1 = t$ and $y_2 = \sqrt{t}$

(b) Use the characteristic solution $y_c = c_1y_1 + c_2y_2$ to find the general solution to the full ODE.

- (7) A mass weighing $1/2$ lb (i.e. mass = $1/64 \text{ lb} \cdot \text{s}^2/\text{ft}$) stretches a spring $1/2$ ft.

(a) Suppose the system has no damping. The mass is initially pulled down $1/2$ ft and released.

(i) Write down the IVP for this system.

(ii) Solve the IVP.

(iii) When does the mass return to the equilibrium position (i.e. $x = 0$).

(b) Now suppose the system has a damping constant of $2 \text{ lb} \cdot \text{s}/\text{ft}$. The mass is initially pushed up $1/2$ ft and released with a downward velocity of $1/2$ ft/s.

(i) Write down the IVP for this system.

(ii) Solve the IVP.

- (8) Please solve the following IVP

$$y'' + 4y = 3 \sin 2t; \quad y(0) = 2, \quad y'(0) = -1.$$