(1) Sketch the direction field for $y' = y(y^2 - 4)$ and state what happens as $t \to \infty$.

(2) Sketch the direction field for $y' = y(y-2)^2$.

(3) Consider the ODE $2t^2y'' + 3ty' - y = 0$.
(a) What is the order of the equation? Is it linear or nonlinear?
(b) Is $y_1 = t^{1/2}$ a solution?
(c) Is $y_2 = t^{-1}$ a solution?
(d) If they are both solutions find the Wronskian of those solutions.

(4) What is the order of the ODE $\frac{d}{dx}\left(x\frac{dy}{dx}\right) = \frac{\ln x}{xy}$. Is it linear or nonlinear?

(5) Solve the IVP $\frac{dy}{dx} = \frac{x}{y(1+x^2)}$, $y(0) = -2$.

(6) Solve the IVP $y' = y^2 - 1$, $y(0) = -2$.

(7) Solve the IVP $y' + y = e^{-t}$, $y(0) = y_0$. Find the value of y_0 such that the solution $y(t)$ reaches its maximum at $t = 4$.

(8) Solve the IVP $ty' + 2y = 4t^2$, $y(1) = 4$.

(9) Find a linear homogeneous constant coefficient ODE that has the roots of its characteristic equation as $r = -2, 3$.

(10) Solve the IVP $y'' - y' - 2y = 0$; $y(0) = \alpha$, $y'(0) = \beta$. What relation between α and β will give us a bounded (for all time) solution?

(11) Consider the IVP: $t(t-4)y'' - 3ty' + 4y = 2$; $y(3) = 0$, $y'(3) = -1$. Determine the longest interval for which the IVP is guaranteed to have a unique solution.

(12) Consider the IVP: $4y'' + 12y' + 9y = 0$; $y(0) = -1$, $y'(0) = \alpha$.
(a) For what α does the solution change signs at $t = 1/2$?
(b) How many times does this solution (for the α above) change signs for $t > 0$?

(13) A tank with a capacity of 6 L initially contains 10 g of salt and 1 L of water. A mixture containing 1 g/L of salt enters the tank at a rate of 3 L/hr. The mixture leaves the tank at a rate of $\frac{1}{2}V(t)$ L/hr, where $V(t)$ is the volume of fluid in the tank (which may be less than the volume of the tank itself).
(a) Formulate the IVP for the volume of fluid in the tank then solve that IVP.
(b) Formulate an ODE for the amount of salt in the tank. Show that this ODE is the same as the ODE for the volume above.

(14) A tank initially contains 120 L of fresh water. A mixture containing a concentration of γ g/L of salt enters the tank at a rate of 2 L/min, and the well-stirred mixture leaves the tank at a rate of 3 L/min.
(a) When will the tank be empty (i.e. come up with an IVP for the volume).
(b) Formulate and then solve an IVP for the amount of salt in the tank.

(15) Please use the method of undetermined coefficients to find the form of the particular solution (**WITHOUT SOLVING FOR CONSTANTS**) of the following ODEs.
(a)

$$y'' + 5y' + 6y = -t + e^{-3t} + te^{-2t} + e^{-3t}\cos t$$

(b)

$$y'' + 3y' + 2y = e^t(t^2+1)\sin(2t) + 3e^{-t}\cos t + 4e^t.$$

(16) Please find the general solution of the ODE: $y'' + 4y' + 4y = t^{-2}e^{-2t};\ t > 0$

(17) Consider the ODE $y'' + 2y' + 2y = \cos t$.
 (a) Please find the general solution.
 (b) What happens to the solution as $t \to \infty$?

(18) Please solve the IVP: $y'' + 4y = 6\sin(4t);\ y(0) = y'(0) = 0$.

(19) Consider the IVP $y'' - 3y' - 4y = t + 2;\ y(0) = 3,\ y'(0) = 0$.
 (a) Please find the solution to the IVP.
 (b) What happens to the solution as $t \to \infty$?

(20) Consider the ODE $2t^2y'' - ty' + y = t\sqrt{t}$.
 (a) Verify the solutions to the homogeneous ODE are $y_1 = t$ and $y_2 = \sqrt{t}$
 (b) Use the characteristic solution $y_c = c_1y_2 + c_2y_2$ to find the general solution to the full ODE.

(21) A mass weighing 1/2 lb (i.e. mass $= 1/64 lb \cdot s^2/ft$) stretches a spring 1/2 ft.
 (a) Suppose the system has no damping. The mass is initially pulled down 1/2 ft and released.
  (i) Write down the IVP for this system.
  (ii) Solve the IVP.
  (iii) When does the mass return to the equilibrium position (i.e. $x = 0$).
 (b) Now suppose the system has a damping constant of $2 lb \cdot s/ft$. The mass is initially pushed up 1/2 ft and released with a downward velocity of 1/2 ft/s.
  (i) Write down the IVP for this system.
  (ii) Solve the IVP.

(22) Please solve the following IVP

$$y'' + 4y = 3\sin 2t;\ y(0) = 2,\ y'(0) = -1.$$

(23) Consider the function

$$g(t) = \begin{cases} e^{-t} & 0 \le t < 1 \\ e^{-3t} + 1 & 1 \le t < 2 \\ 1 & t \ge 2 \end{cases}$$

 (a) Graph the function for $0 \le t \le 3$.
 (b) Write $g(t)$ as unit step functions; i.e. the "u" notation.
 (c) Find the Laplace transform of the function.

(24) Solve the following IVP

$$y'' + 4y' + 8y = 2u_\pi(t) - 2\delta(t - 2\pi);\ y(0) = 2,\ y'(0) = 0$$

(25) Use a convolution to find the Laplace Transform of (Don't integrate)

$$F(s) = \frac{1}{s^3(s^2+1)}.$$

(26) Find the Laplace Transform of

$$f(t) = \int_0^t (t-\tau)^2\cos(2\tau)d\tau$$

(27) Find the inverse Laplace Transform (in closed form) of

$$F(s) = \frac{s^2 - 9}{s^3 + 6s^2 + 9s}.$$

(28) Find the inverse Laplace Transform (in closed form) of

$$G(s) = e^{-s}\frac{s-2}{s^2 + 2s + 2}$$

(29) Find the inverse Laplace Transform of

$$F(s) = \frac{3}{s^2 + 4}$$

(30) Find the inverse Laplace Transform of

$$F(s) = \frac{2s - 3}{s^2 - 4}$$

(31) Find the inverse Laplace Transform of

$$F(s) = \frac{1 - 2s}{s^2 + 2s + 10}$$

(32) Use Laplace Transforms to solve the IVP

$$y^{(4)} - y = 0;\ y(0) = 1,\ y'(0) = 0,\ y''(0) = -2,\ y'''(0) = 0.$$

(33) Use Laplace Transforms to solve the IVP

$$y'' + 4y' = \begin{cases} t & 0 \le t < 1, \\ 0 & t \ge 1 \end{cases};\ y(0) = y'(0) = 0$$

(34) Use Laplace Transforms to solve the IVP

$$y' + ay = e^{\lambda t};\ y(0) = c,$$

with $a \neq 0$. What happens to the solution when $\lambda + a \neq 0$? What about for $\lambda + a = 0$?