Supplementary problems: pg. 64 # 32, 34, 36; pg. 79 # 1, 21, 23, 29

Compulsory problems:

(1) Solve the IVP $y' = 2y - 1;\ y(0) = 1$.

(2) Consider the IVP, where b is a constant,

$$y' = -y + be^{-t};\ y(0) = 0.$$

(a) **[5 pts.]** Solve the IVP.
(b) **[2 pts.]** Show that the solution attains its maximum value at $t = 1$.
(c) **[2 pts.]** For what value of b is this maximum $y = 2$?

(3) Consider the IVP, where a is a constant,

$$ty' + (t+1)y = 2te^{-t},\ t > 0;\ y(1) = a.$$

(a) **[6 pts.]** Solve the IVP.
(b) **[1 pts.]** Show that the solution $y \to 0$ as $t \to \infty$
(c) **[3 pts.]** If $y = 0$ at $t = 2$, what is a?
(d) **[3 pts.]** If the solution y has a critical point at $t = 1/2$, what is a?

(4) Consider two connected tanks: Tank 1 and Tank 2. Initially Tank 1 contains 100 gal of fresh water and Tank 2 100 gal of brine containing 10 lb of salt. Brine containing 0.5 lb/gal of salt is pumped into Tank 1 at 1 gal/min, and the mixture leaves Tank 1 and into Tank 2 and finally out of Tank 2 at the same rate.
(a) **[5 pts.]** Derive the IVP (i.e. ODE + IC) for the salt content in Tank 1.
(b) **[5 pts.]** Derive the IVP for the salt content in Tank 2.
(c) **[4 pts.]** Find the amount of salt in Tank 1 for any time (i.e. solve the IVP).
(d) **[6 pts.]** Find the amount of salt in Tank 2 for any time.

A word on how the grading will work: Let m be the number of supplementary problems, n the number of supplementary problems completed, M the total number of points for the compulsory problems, and N the number of points earned for the compulsory problems. Then your homework score is: $\frac{n}{2m} \cdot M + \left(1 - \frac{n}{2m}\right) \cdot N = N + \frac{n}{2m}(M - N)$. Just be glad it's not a differential equation.