Supplementary problems: # 1, 3, 5, 7

Compulsory problems:

(1) Consider the IVP

$$2y'' + 3y' - 2y = 0;\ y(0) = 1,\ y'(0) = -\beta \text{ (with } \beta > 0).$$

(a) **[7 pts.]** Solve the IVP.
(b) **[7 pts.]** Plot the solution for $\beta = 1$.
(c) **[4 pts.]** Find the minimum of the solution.
(d) **[2 pts.]** Find the smallest (in magnitude) value for $\beta$ for which the solution has no minimum.

(2) Consider the IVP $9y'' + 12y' + 4y = 0$; $y(0) = a > 0$, $y'(0) = -1$.
(a) **[7 pts]** Solve the IVP.
(b) **[3 pts]** Find the critical value of $a$ that separates solutions that become negative from those that are always positive.

(3) Consider the IVP: $y'' + 2y' + 6y = 0$; $y(0) = 2$, $y'(0) = \alpha \geq 0$.
(a) **[6 pts]** Solve the IVP.
(b) **[4 pts]** Find $\alpha$ such that $y = 0$ when $t = 1$.

Your homework raw score is: $\dfrac{n}{2m} \cdot M + \left(1 - \dfrac{n}{2m}\right) \cdot N = N + \dfrac{n}{2m}(M - N)$.