1.1 DESCRIBING A SET

A set is just a collection of elements. For example $\{1, 2, 3\}$, $\{a, b, c\}$, $\{1, 2, 3, a, b, c\}$. Notice that sets need not be in any particular order, but ordering gives us a warm fuzzy feeling. However, we should keep in mind that not all sets can be ordered. In this class we will mainly deal with sets of numbers. Some important subsets of the reals ($\mathbb{R}$) are

$$\mathbb{N} := \{1, 2, 3, \ldots, n, \ldots\}$$
$$\mathbb{Z} := \{0, \pm 1, \pm 2, \ldots, \pm n, \ldots\}$$
$$\mathbb{Q} := \{\frac{m}{n} : n, m \in \mathbb{Z}, n \neq 0\}$$
$$\mathbb{Q}^c := \mathbb{R} \setminus \mathbb{Q}.$$

Complex numbers are also important, but won't be discussed too much in this class.

Now lets look at a few definitions.

Definition 1. A set containing no elements is called the <u>empty set</u>, and is denoted as $\emptyset$ or $\{\}$.

Definition 2. We call the amount of elements in a set A the <u>cardinality</u> of A denoted as $\mathrm{Card}(A)$.

For example, $\mathrm{Card}(\emptyset) = 0$, $\mathrm{Card}(\{1, 2, 3\}) = 3 = \mathrm{Card}(\{a, b, c\})$. For finite sets cardinality is easy, but not so much for infinite ones. We will discuss cardinality in more detail later on.

1.2 SUBSETS

Lets first look at a definition and then do some examples.

Definition 3. A set A is a <u>subset</u> of B if for all $x \in A$, $x \in B$ as well, and this is denoted as $A \subseteq B$. If there is a $y \in B$ such that $y \notin A$, then A is called a <u>proper subset</u> of B, denoted as $A \subset B$.

For example $\{1, 2, 3\} = \{1, 2, 3\}$, $\{1, 2\} \subset \{1, 2, 3\}$, and $\{1, 2, 3, \ldots, ?, \ldots\} \subseteq \{1, 2, 3, \ldots\}$. The possible equal sign is used when there is ambiguity. We may also have sets within sets, for example $\{1\} \subset \{1, 2, \{1\}\}$.

Lets look at Example 1.6 from the book.

Ex 1.6 There we first noted that 1 must be in A and 2 must be in B. We could have 3 and 4 in A, but then we wouldn't have enough elements for B, so lets split them, then we get $A = \{1, 3, 5\}$ and $B = \{2, 4, 5\}$ or $A = \{1, 4, 5\}$ and $B = \{2, 3, 5\}$.

On the real line we are mainly concerned with intervals. We may have closed intervals $[a, b]$, open intervals (a, b), or half open intervals $[a, b)$ or $(a, b]$.

Definition 4. The set of all subsets of A is called its <u>power set</u>, denoted as $P(A)$.

Now lets look at Example 1.8 from the book.

Ex 1.8   (a) $P(A) = \{\emptyset\}$, so $\mathrm{Card}(A) = 0$ and $\mathrm{Card}(P(A)) = 1$.
        (b) $P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$, so $\mathrm{Card}(A) = 2$ and $\mathrm{Card}(P(A)) = 4$.
        (c) $P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$, so $\mathrm{Card}(A) = 3$ and $\mathrm{Card}(P(A)) = 8$.
             So we notice that $\mathrm{Card}(P(A)) = 2^{\mathrm{Card}(A)}$.

Then we did exercise problems 1.2a,d; 1.3a,d; 1.4a,d; 1.12.

1.3 SET OPERATIONS

Here we will go over a bunch of definitions.

Definition 5. The <u>union</u> of sets A and B is $A \cup B := \{x : x \in A \text{ or } x \in B\}$.

Definition 6. The <u>intersection</u> of sets A and B is $A \cap B := \{x : x \in A \text{ and } x \in B\}$.

Definition 7. Two sets A and B are said to be <u>disjoint</u> if $A \cap B = \emptyset$

Definition 8. The <u>complement</u> of $B \subseteq A$ relative to A is $B^c := \{x \in A : x \neq B\}$.

Definition 9. The <u>difference</u> of A and B is $A \setminus B := \{x \in A : x \neq B\}$.

Notice that the complement is a type of difference. For the complement, one set must be a subset of the other.

1.4 INDEX NOTATION

For this section it is best to just look at a few exercises from the book.

1.36 Here $S_1 = [0,3]$, $S_2 = [2,5]$, and $S_3 = [3,6]$, then $\cup_{\alpha \in A} S_\alpha = [0,6]$ and $\cap_{\alpha \in A} S_\alpha = \{3\}$.

1.37 For this one it is not too difficult to go straight to the solution $\cup_{X \in S} = \{1,2,5,0,4,3\}$ and $\cap_{X \in S} = \{2\}$

Then we did exercise problems 1.22, 1.23, 1.38, and 1.39.