Supplementary problems: Sec. 2.5 # 2, 3, 9; 3.1 # 5, 8; 3.2 # 20, 23

Compulsory problems:

(1) **[15 pts.]** Given a set $S \subset \mathbb{R}$, if for all $x \in S$ there is at least one ball $B_r(x)$ such that $B_r(x) \cap S$ is countable, show that S itself is countable. (Don't use the Lindelöf covering theorem because we didn't discuss it and you don't need it.) [Hint: there is a property of $\mathbb{R}$ that makes this proof much easier than $\mathbb{R}^2$.]

(2) **[10 pts.]** If S is uncountable, prove that there is at least one $x \in S$ such that $B_\varepsilon(x) \cap S$ is uncountable for all $\varepsilon > 0$. (Hint: easier than it looks.)

(3) Suppose $\{x_n\} \subset \mathbb{R}$ satisfies $7x_{n+1} = x_n^3 + 6$ for all $n \geq 1$ where $x_1 = 1/2$.
  (a) **[10 pts.]** Prove that $x_n < x_{n+1}$, and
  (b) **[5 pts.]** find its limit. (Recall how to find the limit of recurrence relations; don't use the definition of a limit.)

(4) **[5 pts.]** If $x_n \to x$ and $y_n \to y$, prove that $|x_n - y_n| \to |x - y|$. (You don't have to use the definition of a limit if you don't want to.)

(5) **[10 pts.]** This is # 12 in the book: Show that $(\sqrt{n^2+1} - n) \to 0$. (Use the definition of a limit.)

(6) **[5 pts.]** Prove that every subsequence of a bounded sequence is also bounded; i.e. $|x_m| \leq M \in \mathbb{R} \Rightarrow |x_{k_n}| \leq M \in \mathbb{R}$ for all $\{k_n\}$.

Your homework raw score is: $\dfrac{n}{2m} \cdot M + \left(1 - \dfrac{n}{2m}\right) \cdot N = N + \dfrac{n}{2m}(M - N)$. For this homework, $M = 60$, $m = 7$, N is the number of compulsory problems you get correct, and n is the number of supplementary problems you complete. It should be noted that for the supplementary problems I will be looking for **full completion**, but I won't take off points for mistakes.