Supplementary problems: Sec. 3.3 # 1-3; 3.4 # 3, 9, 15

Compulsory problems:
Since these sequences are nontrivial, please use induction whenever proving increasing/decreasing.

(1) Suppose $0 < x_n < 1$ and $x_{n+1} = 1 - \sqrt{1 - x_n}$ for all $n \geq 1$.
   (a) **[5 pts.]** Prove that $x_n > x_{n+1}$ for all $n \in \mathbb{N}$,
   (b) **[10 pts.]** $x_n \to 0$ (Recall how to find the limit of a recurrence relation.) (Hint: for the limits there is no need to use the formal definition of a limit as long as you can prove the sequences are convergent, which you can do by using part (a).), and
   (c) **[5 pts.]** $x_{n+1}/x_n \to 1/2$. (Use Calc II, not the definition.)

(2) **[10 pts.]** This is # 3.3.4 in the book: If $x_1 = 1$, prove that $x_{n+1} := \sqrt{2 + x_n}$ converges and find its limit.

(3) **[15 pts.]** Consider a sequence defined as $x_1 = 1/2$ and $x_{n+1} := x_n^2$. Prove that it has a limit and find that limit.

(4) **[5 pts.]** Show that the sequence $x_n := \sin \frac{n\pi}{2}$ has convergent subsequences. (Don't use Bolzano–Weierstrass); come up with an example.

(5) **[+10 pts.]** Prove that $(2n)^{1/n}$ is decreasing.

Your homework raw score is: $\dfrac{n}{2m} \cdot M + \left(1 - \dfrac{n}{2m}\right) \cdot N = N + \dfrac{n}{2m}(M - N)$. For this homework, $M = 50$, $m = 6$, N is the number of compulsory problems you get correct, and n is the number of supplementary problems you complete. It should be noted that for the supplementary problems I will be looking for **full completion**, but I won't take off points for mistakes.