Supplementary problems: Sec. 3.4# 18; 3.5 # 5, 9; 3.7 # 3, 7

Compulsory problems:
Since these sequences are nontrivial, please use induction whenever proving increasing/decreasing.

(1) **[15 pts.]** Prove the sequence $\{a_n\}$ converges if $|a_n| < 2$ and $|a_{n+2} - a_{n+1}| \leq \frac{1}{8}|a_{n+1}^2 - a_n^2|$.
(Hint: Can you write $|a_m - a_n|$ in terms of $|a_2 - a_1|$?)
(Do not use the fact that this is a contractive sequence.)

(2) **[15 pts.]** Prove that # 5 in the book $x_n = \sqrt{n}$ is not a contractive sequence. (Hint: Use contradiction.)
(If you compute any limits don't use the definition, just use Calc II techniques.)

(3) **[5 pts.]** Prove (by using partial sums; i.e. different from the proof in the book) that the following series diverges:

$$\sum_{n=1}^{\infty} \frac{1}{n} \tag{1}$$

(4) **[5 pts.]** Give an example of a sequence where $|x_{n+1} - x_n| \to 0$, but for all $N \in \mathbb{N}$ there is an $\varepsilon > 0$ such that there is some $n, m \geq N \Rightarrow |x_m - x_n| \geq \varepsilon$. Show why your example works.

Your homework raw score is: $\frac{n}{2m} \cdot M + \left(1 - \frac{n}{2m}\right) \cdot N = N + \frac{n}{2m}(M - N)$. For this homework, $M = 40$, $m = 5$, N is the number of compulsory problems you get correct, and n is the number of supplementary problems you complete. It should be noted that for the supplementary problems I will be looking for **full completion**, but I won't take off points for mistakes.