Supplementary problems: Sec. 5.2 # 1a, 1b, 1c, 1d; 5.3 # 5; 5.4 # 1, 2, 3a, 3b; 5.6 # 2, 5

Compulsory problems:

(1) Consider the function $f : \mathbb{R} \mapsto (0,1)$ defined as $f(x) = 1/(1+x^2)$. Using the formal definition prove (in order):
  (a) **[5 pts.]** f is continuous at every point on $\mathbb{R}$
  (b) **[10 pts.]** f is uniformly continuous on $\mathbb{R}$

(2) **[10 pts.]** Let $f : [0,1] \mapsto [0,1]$ be continuous. Prove that there is a $c \in [0,1]$ such that $f(c) = c^2$.

(3) **[15 pts.]** Let $f : \mathbb{R} \mapsto \mathbb{R}$ be continuous and suppose that $f(x) = 0$ whenever x is irrational. Prove that $f(x) = 0$ for all $x \in \mathbb{R}$.

(4) For each of the following functions, decide which are bounded above, below, or both on the indicated interval, and which attain their max and min values (Don't write proofs, just briefly state reasoning):
  (a) **[2 pts.]** $f(x) = x^2$ on $(-1,1)$
  (b) **[2 pts.]** $f(x) = x^2$ on $\mathbb{R}$
  (c) **[5 pts.]** On $(-a-1, a+1)$,
$$f(x) = \begin{cases} x^2 & \text{for } x \le a, \\ a+2 & \text{for } x > a; \end{cases}$$
  (d) **[6 pts.]** On $[0,1]$
$$f(x) = \begin{cases} 0 & \text{for } x \text{ irrational}, \\ 1/q & \text{for } p/q \text{ in lowest form}; \end{cases}$$
  (e) **[4 pts.]** On $[0,1]$
$$f(x) = \begin{cases} 0 & \text{for } x \text{ irrational}, \\ -1/q & \text{for } p/q \text{ in lowest form}; \end{cases}$$
  (f) **[6 pts.]** $f(x) = \sin^2(\cos(x) + \sqrt{a+a^2})$ on $[0, a^3]$.

(5) For each of the following polynomial functions, find an integer n such that $f(x) = 0$ for some $x \in [n, n+1]$.
  (a) **[5 pts.]** $f(x) = x^3 - x + 3$
  (b) **[5 pts.]** $f(x) = x^5 + x + 1$

(6) **[15 pts.]** Consider the function $h : (0,\infty) \mapsto (0,1)$ defined as $h(x) = 1/(1+x^2)$. Prove that this is a homeomorphism using the following definition and any theorems we covered. (For the limits/continuity you don't have to use the formal definition)

Definition 1. A map $h : M \mapsto M$, where M is an manifold (interval), is said to be a homeomorphism if it is bijective and bicontinuous (continuous with a continuous inverse).

Your homework raw score is: $\dfrac{n}{2m} \cdot M + \left(1 - \dfrac{n}{2m}\right) \cdot N = N + \dfrac{n}{2m}(M - N)$. For this homework, $M = 90$, $m = 11$, N is the number of compulsory problems you get correct, and n is the number of supplementary problems you complete. It should be noted that for the supplementary problems I will be looking for **full completion**, but I won't take off points for mistakes.